

# Common Core Math -Concrete, Representational, Abstract-



2017 Fall Institute

## Facilitator



Valencia Carmichael, UNCG, [vwarmichael@uncg.edu](mailto:vwarmichael@uncg.edu)

LaTonya Gaines-Montgomery, UNCC, [legaines@uncc.edu](mailto:legaines@uncc.edu)

Kim Ramadan, UNCC, [kheintsc@uncc.edu](mailto:kheintsc@uncc.edu)

## Objectives



At the end of this session you will be able to:

- describe the components of concrete-representational-abstract (CRA) instruction
- create a lesson supported by the CRA sequence
- relate the benefits of CRA

NCNTSP.NORTHCAROLINA.EDU

## True or False



1. Manipulatives may be used before or after a procedure is learned with generally equal success.
2. Manipulatives are more useful with less-experienced students than more-experienced students.
3. Almost any manipulative can be used to teach any mathematical concept.

NCNTSP.NORTHCAROLINA.EDU

## Video

Concrete-Representational-Abstract Instructional Model

**Concrete-Representational-Abstract**  
A Three Stage Instructional Approach to Teaching Math Concepts

1:48

0.04 / 2.59

NCNTSP.NORTHCAROLINA.EDU

## Research Based Concept

- Teaching equivalent fraction concepts and procedures (Butler et al., 2003)
  - Fraction circles and beans
  - Coloring portion of items to make fraction
  - Abstract symbols and algorithm
  
- Teaching multiplication facts and related word problems (Morin and Miller, 1998)
  - Paper plates and wooden blocks
  - Pictures of containers and items
  - Numbers

NCNTSP.NORTHCAROLINA.EDU

## Explaining C-R-A



- Concrete or hands-on instruction that involves the manipulation of objects
- Representational stage, with different levels including pictures, technology, or tally marks
- Abstract stage, involving the use of numerals and operational symbols to represent the previous levels

NCNTEP.NORTHCAROLINA.EDU

## CRA Principals



The CRA sequence should be very *explicit* and *systematic*, with multiple opportunities for both *guided practice* and *independent practice*.

NCNTEP.NORTHCAROLINA.EDU

## Definition of Concrete



- The use of manipulatives or models that represent the concept being studied is critical.
- It is the “doing” step of instruction.
- Students must be fluent in manipulation of the concrete materials and must be able to explain the connection to the skill.
- Howell and Barnhart (1992) identified six steps in teaching from concrete to representational levels.

NCNTSP.NORTHCAROLINA.EDU

## Stage 1: Concrete



- Step 1: Free exploration
- Step 2: Purposeful exploration
- Step 3: Number cards
- Step 4: Number sentence
- Step 5: Mathematical word problems
- Step 6: Verbal explanation

NCNTSP.NORTHCAROLINA.EDU

### Purposeful Exploration

11

### Step 3: Number Cards

6 + 3 = 9

12

  
**3 stars + 4 stars = 7 stars**

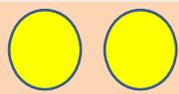


NCNTSP.NORTHCAROLINA.EDU 13

**Step 5: Mathematical Word Problems**



There are three green buttons in a box. There are also two yellow buttons in the box. How many buttons are in the box altogether?

3 2

+ = 5

NCNTSP.NORTHCAROLINA.EDU 14

## Step 6: Verbal Explanation



- Students should verbalize what they did to solve the problem and why.
- Students should be taught to mastery level for each step before moving to the next step.

15

NCNTSP.NORTHCAROLINA.EDU

## Activity



- Working with a partner, teach students to solve a subtraction, addition, or multiplication problem by using manipulatives.
- Keep the steps in mind:
  - Step 1: Free exploration
  - Step 2: Purposeful exploration
  - Step 3: Number cards
  - Step 4: Number sentence
  - Step 5: Mathematical word problems
  - Step 6: Verbal explanation

16

NCNTSP.NORTHCAROLINA.EDU

## Definition of Representational

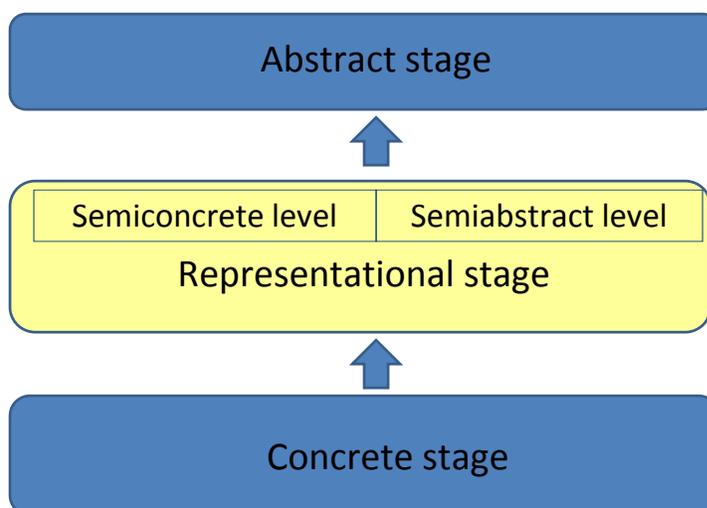


- Replacing concrete manipulative objects with pictures or drawings
- The “seeing” step of instruction
- Division into a semiconcrete level and a semiabstract level
- Six steps in the representational stage (Howell and Barnhart, 1992)

NCNTSP.NORTHCAROLINA.EDU

17

### Stages and Levels



(Howell and Barnhart, 1992)

## Representational

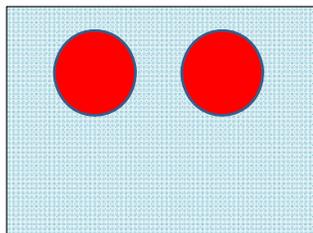
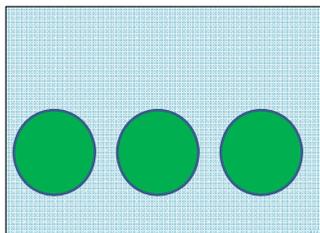


- Step 1: Picture representation cards
- Step 2: Number sentences (semiconcrete)
- Step 3: Mathematical word problems
- Step 4: Graphic representation
- Step 5: Number sentences (semiabstract)
- Step 6: Mathematical word problems

(Howell and Barnhart, 1992)

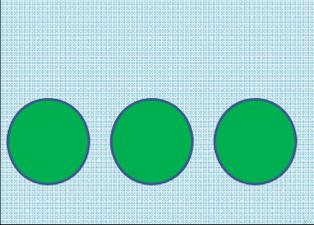
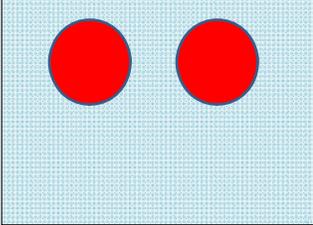
19

## Step 1: Picture Representation Cards



20

### Step 2: Number Sentence

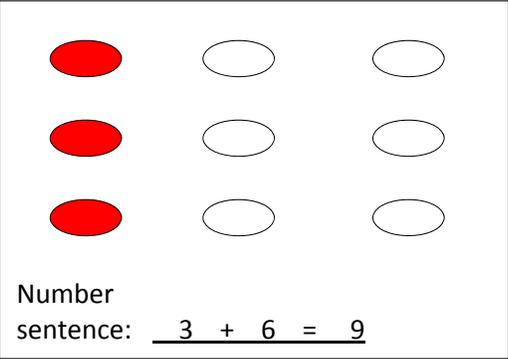
  
  


$3+2=5$

21

NCNTSP.NORTHCAROLINA.EDU

### Step 3: Mathematical Word Problems

Number sentence: 3 + 6 = 9

22

NCNTSP.NORTHCAROLINA.EDU

## Step 4: Graphic Representation



Red Ovals	White Ovals	Total
///	////	<del>///</del> //

NCNTSP.NORTHCAROLINA.EDU

23

## Step 5: Number Sense



Red Ovals	White Ovals	Total
///	////	<del>///</del> //
<div style="display: flex; justify-content: center; gap: 10px; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px; display: inline-block;">3</div> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px; display: inline-block;">+</div> <div style="border: 1px solid black; padding: 2px 5px; display: inline-block;">4</div> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px; display: inline-block;">=</div> <div style="border: 1px solid black; padding: 2px 5px; display: inline-block;">7</div> </div>		

NCNTSP.NORTHCAROLINA.EDU

24

## Step 6: Mathematical Word Problems



Jackie has \$4 allowance left. Her mom gives her \$12 more today. How much allowance does she have altogether?

Allowance	Allowance	Total
////	### ## //	### ## ### /
4	+	12 = 16

25

NCNTSP.NORTHCAROLINA.EDU

## Definition of Abstract



- The “symbolic” step of instruction
- Written mathematical numbers are used to represent concepts or skills
- Thinking strategy to solve word problems, with five questions to ask (Howell and Barnhart, 1992)

26

NCNTSP.NORTHCAROLINA.EDU

## Stage 3: Abstract



Five questions used as procedural prompts that help students become independent problem solvers:

1. What is the question?
2. What are the numbers in the problem?
3. What do I need to do with the numbers?
4. What is the answer?
5. How can I check the answer?

(Howell and Barnhart, 1992)

27

NCNTSP.NORTHCAROLINA.EDU

## Guidelines for using Manipulatives



1. Select manipulatives that clearly illustrate the concept.
2. Use a variety of manipulatives that illustrate the concept.
3. Provides verbal explanations while incorporating the manipulatives.
4. Provide multiple opportunities for guided practice.
5. Encourage the use of manipulatives and strategies across settings.
6. Plan a graduated instructional sequence that will allow transition from concrete to symbolic representation.

28

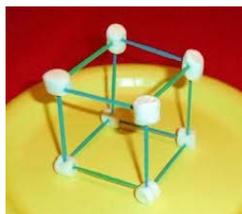
NCNTSP.NORTHCAROLINA.EDU

# This or That?



Expensive	Cost Effective

# Meaningful Manipulatives



## Manipulatives



NCNTSP.NORTHCAROLINA.EDU

## Lesson Planning



- Look at the standard for your grade level.
- Working with a partner, plan an appropriate math lesson.
- Make sure you include the CRA approach and manipulatives.

NCNTSP.NORTHCAROLINA.EDU

# Closure



Think about what you learned in this session, and relate it to your classroom. Write down some ideas of what you want to start using in your classroom on the chart paper provided. Be prepared to share 1-2 responses with the entire group.

## Explore Resources/Thoughts/Question, Answer

